# Microwave Characteristics of High $T_c$ Superconducting Coplanar Waveguide Resonator

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***Abstract*—A theoretical formulation has been developed to calculate the coupling coefficient, London penetration depth, and surface resistance of a coplanar waveguide resonator fabricated from films of superconducting YBCO material. Experimental data of the reflection coefficient as a function of temperature and frequency agree reasonably well with calculations. The formulation is of sufficient generality to be applicable to other guided structures.**

## Introduction

THE DISCOVERY of high-$T_c$ superconductor oxides has made possible a new class of microwave devices operating at temperatures considerably above liquid helium temperatures. Passive microwave components are among the first practical devices fabricated from these high-$T_c$ materials in which the surface resistance $R_s$ is a measure of the device performance at any frequency. Also, knowledge of the London penetration depth $\lambda$ of a superconductor provides further information about microscopic properties of the superconducting state. For example, the density of Cooper pairs may be related to $\lambda$. Thus, the two important parameters which characterize the microwave properties of superconducting films are $R_s$ and $\lambda$. In this paper a formalism in which $R_s$ and $\lambda$ may be deduced from microwave reflection data of a superconducting resonator has been developed. The object of this paper is to demonstrate the technique of this analysis in deducing fundamental parameters of superconducting films and obtaining circuit parameters of the device. We believe that the analysis will serve as a future guide to the design of passive linear devices. Although a coplanar waveguide (CPW) resonator is considered here, our calculations may be applied to other linear resonators, e.g., stripline, microstrip line, slot line, etc. One advantage of a CPW resonator over a microstrip or stripline is that both the ground and conducting planes are in the same film plane. Other designs have the ground plane physically removed from the conducting plane. A CPW resonator was fabricated from YBCO thin film deposited on an MgO substrate by laser deposition techniques. The deduced $\lambda$ at 0 K was 0.179 $\mu$m and $R_s$ was 10.3 m$\Omega$, which is about one third of that of copper at 14.6 GHz at room temperature.

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## Theoretical Formulation

The complex conductivity of a superconductor,

$$\sigma = \sigma_1 - i\sigma_2,$$

can be formulated using the two fluid model. The resistive part of the conductivity, $\sigma_1$, may arise from normal electron conduction within nonsuperconducting grains and scattering from grain boundaries, flux vibration at pinning centers, and normal electron conduction due to thermal agitation in the superconducting state. The temperature dependence of $\sigma_1$ in a high-$T_c$ ceramic superconductor is therefore very complicated. The reactive part of the conductivity, $-i\sigma_2$, arises from the lossless motion of the superconducting carriers which may be derived directly from the Lorentz-force equation as [1]

$$\sigma_2 = \frac{1}{\omega\mu_o\lambda^2},$$

where $\lambda$ is the London penetration depth defined by

$$\lambda = \sqrt{\frac{m_q}{\mu_o q^2 n_s}},$$

and $m_q$, $q$, and $n_s$ are the mass, charge, and volume density of the superconducting carriers, respectively. The surface impedance, $Z_s$, and penetration distance, $\delta$, are defined in the usual way as [2]

$$Z_s = R_s + iX_s = \sqrt{i\omega\mu_o/\sigma}$$

$$\delta = \sqrt{\frac{2}{\omega\mu_o|\sigma|}}.$$

Note that $\delta$ approaches the classical skin depth or the London penetration depth in the limits of high $\sigma_1$ or high $\sigma_2$, respectively. In the latter case, for $\sigma_2 >> \sigma_1$, one may write

$$R_s = \frac{1}{2}\omega^2\mu_o^2\lambda^3\sigma_1, \tag{1}$$

$$X_s = \omega\mu_o\lambda. \tag{2}$$

Empirically, we scale $\lambda$ with temperature as follows [3]

$$\lambda = \lambda_o/\sqrt{1 - (T/T_c)^4}, \tag{3}$$

where $\lambda_o$ is the penetration depth at 0 K and $T_c$ is the transition temperature of superconductivity.

For a coplanar waveguide (CPW) line the characteristic impedance, $Z_o$, can be evaluated by using the following equation [4]

$$Z_o(\text{Ohm}) = \frac{30}{\sqrt{\epsilon_{re}}}\left[\ln\left(2\frac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)\right], \tag{4}$$

where

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2}\left\{\tanh\left(1.785 \log\frac{h}{W} + 1.75\right) + \frac{kW}{h}[0.04 - 0.7k + 0.01(1 - 0.1\epsilon_r) \cdot (0.25 + k)]\right\},$$

$$k' = (1 - k^2)^{1/2},$$

and $k$ is defined as

$$k = \frac{S}{S + 2W}.$$

Here $S$, $W$, $h$, and $\epsilon_r$ are, respectively, the central-strip width, distance between the central strip and ground electrodes, thickness, and dielectric constant of the dielectric slab, see Fig. 1. Note that the above formulae can only apply to a CPW line for which $0 \le k \le 0.707$ and the conductor thickness, $t$, approaches zero [4]. For finite $t$ the above formulae can still be applied provided that modified $S$ and $W$ values are used:

$$S \to S + \Delta,$$

$$W \to W - \Delta,$$

and $\Delta$ is given by

$$\Delta = \frac{1.25t}{\pi}\left[1 + \ln\frac{4\pi S}{t}\right]. \tag{5}$$

Denote $R_1^0(\approx 0)$, $L_1^0$, and $C_1^0$ as, respectively, the (series) resistance, (series) inductance, and (shunt) capacitance per unit length of the line determined for a perfect conductor. The real distributed elements of the line, $R_1$, $L_1$, and $C_1(\approx C_1^0)$, should include the kinetic as well as dissipative contributions of the superconducting carriers and normal electrons within the penetration depth, $\delta$. The incremental inductance and resistance can be estimated through recessing the perfect conducting surfaces a distance $\delta$ into the conductor walls. Since $\delta \approx \lambda(\lesssim t)$ for $\sigma_2 >> \sigma_1$, one may write

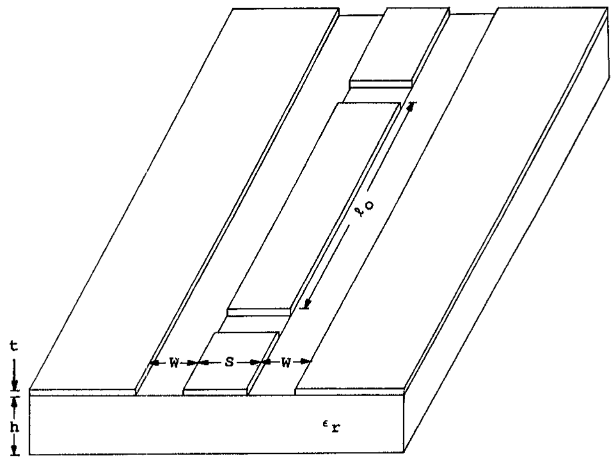


Fig. 1. CPW resonator configuration.

$$L_1 = L_1^0 + \lambda\frac{\partial L_1^0}{\partial n}, \tag{6a}$$

$$R_1 = \left(\frac{R_s}{X_s}\right)\left(\lambda\frac{\partial \omega L_1^0}{\partial n}\right), \tag{6b}$$

where

$$\frac{\partial}{\partial n} = 2\left[\frac{\partial}{\partial W} - \frac{\partial}{\partial S} - \frac{\partial}{\partial t}\right].$$

$L_1^0$ is related to the characteristic impedance of the line with the dielectric substrate replaced by air, $Z_o^a$, as

$$L_1^0 = Z_o^a/c,$$

$Z_o^a$ is related to $Z_o$ by

$$Z_o = Z_o^a/\sqrt{\epsilon_{re}},$$

and $c$ is the speed of light in vacuum. Note that (6a) and (6b) are valid only when $\lambda << t$. When $\lambda$ approaches $t$, the rf field can penetrate through the film and the induced kinetic inductance of the line will increase abnormally. For the most general cases, even for $\lambda > t$, one may still use (6a) and (6b) provided that parameter $\lambda$ in these two equations is replaced by $\lambda_e$ defined as [5]

$$\lambda_e = \lambda \coth(t/\lambda). \tag{6c}$$

One is therefore required to evaluate the following derivative

$$\begin{aligned}\frac{1}{Z_o}\left(\frac{\partial Z_o}{\partial n}\right) &= \frac{2}{Z_o}\left(\frac{\partial}{\partial W} - \frac{\partial}{\partial S} - \frac{\partial}{\partial t}\right)Z_o \\ &= \frac{2}{Z_o}\left(1 + \frac{\partial \Delta}{\partial t}\right)\left(\frac{\partial}{\partial W} - \frac{\partial}{\partial S}\right)Z_o \\ &= \frac{2}{Z_o}\left(1 + \frac{\partial \Delta}{\partial t}\right)\left(\frac{\partial k}{\partial W} - \frac{\partial k}{\partial S}\right)\frac{dk'}{dk}\frac{d}{dk'}Z_o(k') \\ &= \frac{4}{S}\left(\frac{k}{\sqrt{k'}}\right)^3 \frac{1 + W/S}{1 - k'}\left(1 + \frac{1.25}{\pi}\ln\frac{4\pi S}{t}\right)\Big/\ln\left(2\frac{1 + \sqrt{k'}}{1 - \sqrt{k'}}\right). \end{aligned} \tag{7}$$

The conductor attenuation constant, $\alpha_c$, and the phase propagation constant, $\beta$, can be then expressed as

$$\alpha_c = \frac{R_1}{2Z_o} = \frac{R_s\sqrt{\epsilon_{re}}}{120\pi}\left(\frac{1}{Z_o}\frac{\partial Z_o}{\partial n}\right), \tag{8a}$$

$$\beta = \omega\sqrt{L_1 C_1} = \frac{\omega\sqrt{\epsilon_{re}}}{c}\left[1 + \frac{\lambda_e}{2}\left(\frac{1}{Z_o}\frac{\partial Z_o}{\partial n}\right)\right]. \tag{8b}$$

From (8b) the resonant frequency of a CPW resonator shown in Fig. 1 can be therefore expressed as

$$f_1 = f_o - \delta f$$

where $\delta f$ denotes the shift of resonant frequency due to the kinetic inductance arising from the lossless motion of the superconducting carriers within the London penetration depth:

$$\frac{\delta f}{f_o} = \frac{\lambda_e}{2}\left(\frac{1}{Z_o}\frac{\partial Z_o}{\partial n}\right). \tag{9}$$

Note that in deriving (9) we have assumed $\delta f << f$, i.e. the kinetic inductance of the line is much smaller than its normal distributed inductance. The total quality factor, $Q_T$, of the resonator is

$$Q_T^{-1} = Q_c^{-1} + Q_d^{-1}$$

and $Q_c$ and $Q_d$ represent the quality factors associated with conductor loss and dielectric loss of the line, respectively. $Q_c$ and $Q_d$ can be expressed as

$$Q_d = 1/\tan\delta, \tag{10}$$

$$Q_c = \frac{\beta}{2\alpha_c} = \frac{240\pi^2 f_o}{cR_s}\left(\frac{1}{Z_o}\frac{\partial Z_o}{\partial n}\right)^{-1}, \tag{11}$$

and tan $\delta$ denotes the loss tangent of the dielectric substrate. Losses due to radiation and surface propagating waves have been tentatively ignored at this moment. However, as relatively low $Q$-values were measured for a CPW resonator compared with those of YBCO cavities, one may conclude that radiation and surface waves constitute the most important loss mechanisms for an open resonator structure. This has been generally demonstrated for microstrip patch antennas. We note that (9) to (11) can be equally used for other linear-resonator configurations, provided that the characteristic impedance of the line can be explicitly calculated from the line geometry.

We approximate the CPW resonator in the vicinity of resonance as an RCL circuit [6] shown in Fig. 2. The input impedance, $Z_i$, can be expressed as

$$Z_i = \kappa Z_o\Big/\left[1 - iQ_o\left(\frac{f}{f_1} - \frac{f_1}{f}\right)\right], \tag{12a}$$

where resistances $Z_o$ and $R_o$ are related to conductances $Y_o$ and $G_o$ as

$$Z_o = 1/Y_o,$$

$$R_o = 1/G_o,$$

$f_1$ denotes the resonant frequency given by

$$f_1 = 1/(2\pi\sqrt{L_o C_o}), \tag{12b}$$

and $Q_o$ is the intrinsic quality factor of the resonator given by

$$Q_o = 2\pi R_o C_o f_1. \tag{12c}$$

$\kappa$ in (12) defines the coupling coefficient as

$$\kappa \equiv n^2 R_o/Z_o, \tag{12d}$$

which measures the loading of the coupled circuit to the source. Critical coupling occurs when $\kappa = 1$, while $\kappa$ smaller (larger) than 1 corresponds to the case of undercoupling (overcoupling). Reflection magnitude is therefore

$$\begin{aligned}|\Gamma_i| &= \left|\frac{Z_o - Z_i}{Z_o + Z_i}\right| \\ &= \left[\frac{(\kappa - 1)^2 + 4Q_o(f/f_1 - 1)^2}{(\kappa + 1)^2 + 4Q_o(f/f_1 - 1)^2}\right]^{1/2}. \end{aligned} \tag{13}$$

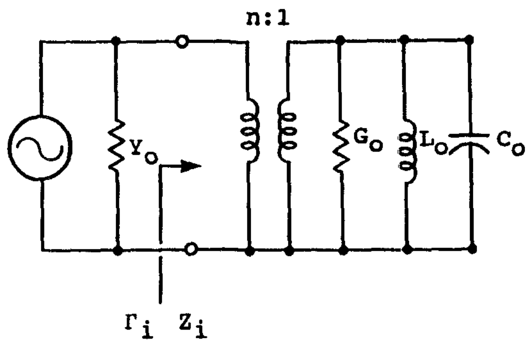



Fig. 2. Equivalent circuit for coupling a resonator near resonance.

We note that $Q_o$ defined in terms of lumped elements shown in Fig. 2 can realistically represent the intrinsic $Q$ of the CPW resonator shown in Fig. 1 only if the loading of the resonator by the output port is negligible. In the following measurements we have used a very large (output) gap in the YBCO CPW resonator design which effectively isolated the resonator from coupling into the output port. In essence, the device we have used was actually a one-port cavity.

## Experiments

High-quality oriented thin films of $YBa_2Cu_3O_{7-\delta}$ were fabricated using pulsed-laser-deposition techniques. The films were deposited on MgO (100) substrates at 750°C, and then quenched *in situ*. The film used in CPW resonator fabrication had a thickness of 0.5 $\mu$m which showed a sharp transition temperature around 86 K with $\Delta T \approx$ 0.5 K. The film was then patterned utilizing a traditional photolithographic technique. Two coats of Shipley 1813 positive photoresist were applied to the film which was then illuminated under a photo mask with a uv light (wavelength 4360 Å). The photoresist was developed in Shipley Microposit (MF319) and was etched in ethylenediamine tetra-acetic (EDTA) acid solution. The produced CPW resonator had the following dimensions: $W$ = 0.4 mm, $S$ = 0.2 mm, $h$ = 1 mm, $t$ = 0.5 $\mu$m, $\epsilon_r$ = 10, and the (input) coupling gap and the length of resonator, $l_o$, were 0.2 mm and 4.9 mm, respectively. The output gap was 3.5 mm, which effectively isolated the resonator from its output port.

The resonator was inserted in a low-temperature dewar whose temperature was varied from 60 K to 90 K. Reflection data of the resonator were collected using a Network Analyzer HP8510B which was connected to an HP computer. The input microwave power was fixed to −10 dBm during the measurements. Experimental data at each temperature were then fitted with least-square-errors to the function defined in (13). In this manner the temperature-dependent parameters, $f_o$, $\kappa$, and $Q_o$ were determined. Fig. 3 shows a typical fit of the two curves in which curve 1) is the experimental $\Gamma_i$ measured at $T$ = 81.5 K and curve 2) represents a curve calculated from (13). $f_o$, $\kappa$, and $Q_o$ are plotted as functions of temperature as shown in Figs. 4 to 6, respectively.

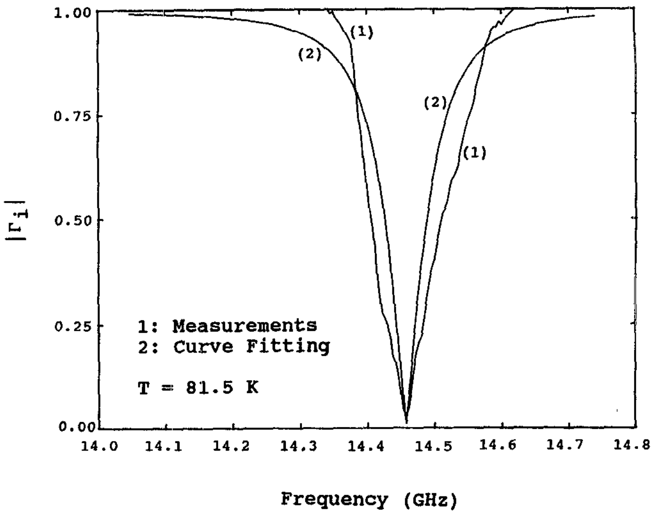



Fig. 3. Reflection coefficient $|\Gamma_i|$ versus frequency.

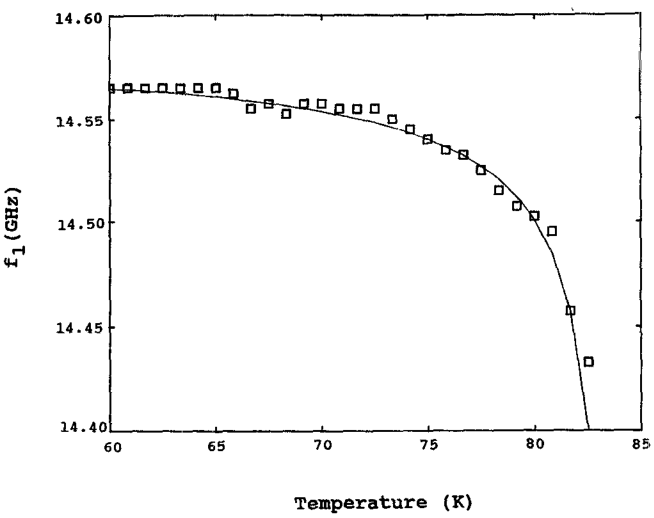



Fig. 4. Resonant frequency $f_1$ as a function of temperature. Solid line represents least-square-fit to experimental data (□'s).

The solid line in Fig. 4 represents a least-square fit to the experimental $f_o$ values in which (9) and the temperature dependence (3) have been assumed. This determined $f_o$, $T_c$, and $\lambda_o([1/Z_o][\partial Z_o/\partial n])$ as

$$f_o = 14.567 \text{ GHz,} \tag{14a}$$

$$T_c = 83{,}7 \text{ K}, \tag{14b}$$

$$\lambda_o\left(\frac{1}{Z_o}\frac{\partial Z_o}{\partial n}\right) = 3.23 \times 10^{-3}. \tag{14c}$$

The transition temperature deduced from our data showed a slight degradation in $T_c$ in the order of 2 K during the patterning of the thin film resonator. The value of $(1/Z_o)(\partial Z_o/\partial n)$ calculated from (7) is 0.018 $\mu m^{-1}$, which

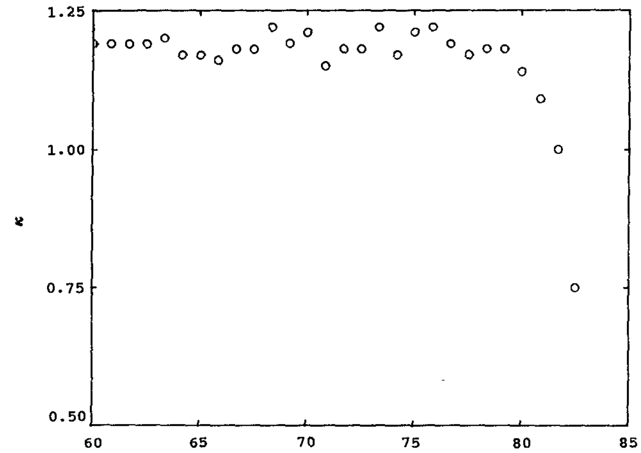



Fig. 5. Coupling coefficient $\kappa$ as a function of temperature.

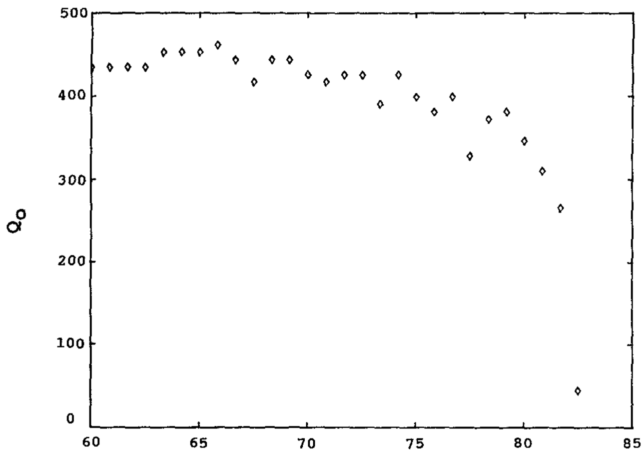



Fig. 6. Intrinsic quality factor $Q_o$ as a function of temperature.

implies that the London penetration depth at 0 K is

$$\lambda_o = 0.179 \ \mu\text{m}.$$

Fig. 5 shows the coupling coefficient $\kappa$ as a function of temperature. $\kappa$ measures the relative strength of the transferred impedance of the resonator to that of the feeding line (50 Ω), see (12d). When the temperature was slightly below $T_c$, the resonator was undercoupled ($\kappa = 0.75$), and when the temperature was lowered well below $T_c$, the resonator became overcoupled ($\kappa = 1.24$). The resonator was critically coupled at $T = 81.5$ K. $\kappa$ increases as temperature decreases, since $R_o$ increases with decreasing temperature. In Fig. 6 $Q_o$ is plotted as a function of temperature. At 60 K $Q_o$ reached a maximum value of 435, which corresponded to a surface resistance $R_s = 10.3$ mΩ. This value is about one third of that of copper at the same frequency at room temperature (31.4 mΩ). Note that in the above analysis we have ignored the dielectric loss of the resonator due to the smallness of the loss tangent of the substrate MgO: tan $\delta = 0.0005$ for MgO for $T < 100$ K. Low $Q$-values measured in a CPW resonator indicate that radiation and surface-propagating wave losses might be sigificant in such an open structure.

## Conclusions

We have developed a calculational scheme to analyze linear resonators in general. From this analysis the rf-field penetration depth and surface resistance of the conducting electrodes can be accurately deduced. As a test case high-$T_c$ superconducting films were used to fabricate CPW resonators. The London penetration depth of 0.179 $\mu$m deduced from this work is in agreement with literature as measured by other techniques [7]. Since conduction is in the ab plane, the penetration depth $\lambda$ is associated with the direction of the microwave electric field in the film plane. This should be compared with a value of 3600 Å for the case of conduction along the $c$-axis [7]. The coupling coefficient of the superconducting resonator was temperature dependent, reaching critical-coupling at a temperature slightly below $T_c$. Finally, the surface resistance of the film was measured to be 10.3 mΩ at 14.567 GHz, which is compared with the lowest reported $R_s$ value of 16 $\mu\Omega$ at 4 K and 10 GHz for a single crystal YBCO sample [8]. For single crystal YBCO, $R_s$ is roughly two orders of magnitude lower than for polycrystalline materials. The maximum theoretical $Q$ that can be achieved in our resonator was limited by $Q_d \approx 2000$. The measured $Q$ was actually $\leq 500$. The difference is accounted for by ohmic and radiation/surface-wave losses. Ohmic losses may be related to fluxoid motion in the superconductor. Measured $Q$'s exceeding 1000 have been observed for linear microstrip resonators using $LaAlO_3$ substrates [5], [9].

## References


[1] H. How, A. Widom, and C. Vittoria, "Model for the A.C. Hall effect of high-$T_c$ superconductors," *J. Superconductivity*, vol. 2, no. 4, p. 479, 1989.

[2] J. D. Jackson, *Classical Electrodynamics*. New York: Wiley, 1974, ch. 7.

[3] J. R. Schrieffer, *Theory of Superconductivity*. New York: W. A. Benjamin, 1964, ch. 1.

[4] K. C. Gupta, R. Gard, and I. J. Bahl, *Microstrip Lines ad Slotlines*. Norwood, MA: Artech House, 1979, ch. 7.

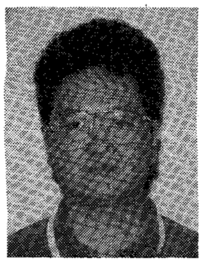
[5] S. M. Anlage, H. Sze, H. J. Snortland, S. Tahara, B. Langley, C. Eom, M. R. Beasley, and R. Taber, "Measurements of the magnetic penetration depth in $YBa_2Cu_3O_{7-\delta}$ thin films by the microstrip resonator technique," *Appl. Phys. Lett.*, vol. 54, no. 26, p. 2710, 1989.

[6] D. Kajfez and E. T. Hwan, "$Q$-factor measurement with network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, 7, pp. 666-670, 1984.

[7] R. Karim, C. Vittoria, A. Widom, D. B. Chrisey, and J. S. Horwitz, "Magnetically modulated microwave absorption measurement of the penetration depth in a polycrystalline $YBa_2Cu_3O_{7-x}$ thin film," *J. Appl. Phys.*, vol. 69, no. 1, 1991.

[8] S. S. Laderman, R. C. Taber, R. D. Jacowitz, J. L. Moll, C. B. Eom, T. L. Hylton, A. F. Marshall, T. H. Geballe, and M. R. Beasley, "Resistive loss at 10 GHz in c-axis aligned in situ growth $YBa_2Cu_3O_7$ films," *Phys. Rev. B*, vol. 43, no. 4A, p. 2922, 1991.

[9] J. H. Takemoto, C. M. Jackson, R. Hu, J. F. Burch, K. P. Daly, and R. W. Simon, "Microstrip resonators and filters using high-$T_c$ superconducting thin films on $LaAlO_3$," *IEEE Trans. Magn.*, vol. 27, no. 2, p. 2549, 1991.



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